Quantum critical behavior near a density-wave instability in an isotropic Fermi liquid

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(Dated: cond-mat/0405344, v.2 8 February 2005, Phys. Rev. Lett. **94**, 046404 (2005))

We study the quantum critical behavior in an isotropic Fermi liquid in the vicinity of a zero-temperature density-wave transition at a finite wave vector q_c. We show that, near the transition, the Landau damping of the soft bosonic mode yields a crossover in the fermionic self-energy from $\Sigma(k,\omega) \approx \Sigma(k)$ to $\Sigma(k,\omega) \approx \Sigma(\omega)$, where k and ω are momentum and frequency. Because of this self-generated locality, the fermionic effective mass diverges right at the quantum critical point, not before, i.e., the Fermi liquid survives up to the critical point.

PACS numbers: 71.10.Ay 71.10.Ca 71.10.Hf 71.18.+y

Introduction. An isotropic Fermi liquid may experience various intrinsic quantum instabilities. They are characterized by divergence of the corresponding static susceptibility and emergence of a gapless bosonic mode in the collective, two-particle excitation spectrum. The instability point at zero temperature, occurring at a particular value of a control parameter such as electron concentration, is called the quantum critical point (QCP). Near QCP, the interaction between the soft bosonic mode and low-energy fermions often leads to singular behavior of the fermionic self-energy $\Sigma(k,\omega)$ and divergence of the fermionic effective mass m^*. A well-known example is the divergence of m^* near ferromagnetic instability [1], which occurs at the wave vector $q = 0$.

In this paper, we study the divergence of m^* in an isotropic Fermi liquid near a zero-temperature charge- or spin-density-wave instability occurring at a nonzero wave vector $q_c \leq 2k_F$, where k_F is the Fermi momentum. We argue that, for $q_c \neq 0$, the behavior of m^* near QCP is rather tricky, and the analysis requires extra care.

We consider a model in which fermions ψ_k interact by exchanging a soft bosonic mode $V(q)$

$$\mathcal{H}_{\rm int} = -\sum_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'} \psi^\dagger_{\boldsymbol{k}+\boldsymbol{q}} \psi^\dagger_{\boldsymbol{k}'-\boldsymbol{q}} V(\boldsymbol{q})\, \psi_{\boldsymbol{k}'} \psi_{\boldsymbol{k}}. \tag{1}$$

The soft bosonic mode is peaked at the wave vector q_c and can be in either spin or charge channel

$$V(q) \approx \frac{g}{\xi^{-2} + (|\boldsymbol{q}| - q_c)^2}. \tag{2}$$

Here g is an effective interaction constant, and ξ is the correlation length, which diverges at QCP as a function of electron concentration or pressure.

Interaction between soft bosonic modes (2) was studied by Brazovskii [2] in the context of a crystallization transition in an isotropic liquid. Dyugaev [3] applied the model (1) and (2) to explain enhancement of the effective mass and specific heat in liquid ^{3}He, arguing that ^{3}He is close to a spin-density-wave transition. Ref. [4] utilized the Brazovskii model to describe a magnetic transition in MnSi, where a finite q_c is likely caused by the Dzyaloshinskii-Moriya interaction. The model (1) and (2) also applies to itinerant electrons in the vicinity of a ferromagnetic instability – a small but finite q_c appears there as a result of an effective long-range interaction between fermions due to the $2k_F$ Kohn anomaly [5]. Refs. [6, 7, 8] proposed that the model (1) and (2) can explain the enhancement and possible divergence of the effective mass observed experimentally in the two-dimensional electron gas (2DEG) [9], as well as in ^{3}He films [10]. In the scenario of Refs. [6, 7, 8], the instability at q_c develops as a precursor to the Wigner crystal in 2DEG or to the crystallization transition in ^{3}He films.

The effective mass m^* is extracted from the fermionic self-energy $\Sigma(k,\omega)$ defined by the Dyson equation $G^{-1}(k,\omega) = i\omega - \varepsilon_k - \Sigma(k,\omega)$, where $G(k,\omega)$ is the fermion Green's function, and $\varepsilon_k = v_F(k - k_F)$ is the bare fermion dispersion counted from the chemical potential. The derivatives of $\Sigma(k,\omega)$ determine the renormalization factor $Z^{-1} = 1 + i\partial_\omega\Sigma$ and the effective mass $m^*/k_F = 1/v_F^* = Z^{-1}(v_F + \partial_k\Sigma)^{-1}$, where v_F and v_F^* are the bare and renormalized Fermi velocities. Divergence of m^* can be caused either by $i\partial_\omega\Sigma \to \infty$ (and, hence, $Z \to 0$), or by $\partial_k\Sigma \to -v_F$. In the former case, m^* would diverge at QCP, but not earlier, while in the latter case, the divergence of m^* would generally occur at a finite distance from QCP. There exist other scenarios [11] for the divergence of m^*, which do not invoke a density-wave transition, but we will not discuss them here.

The interplay between $\partial_\omega\Sigma$ and $\partial_k\Sigma$ depends on whether $\Sigma(k,\omega)$ predominantly depends on momentum k or on frequency ω. The two alternative scenarios for the model of Eqs. (1) and (2) where Σ depends only on k or only on ω were advocated in Refs. [6, 7, 12] and Refs. [3, 4], correspondingly. In this paper, we show that the behavior of Σ in an isotropic Fermi liquid near a density-wave transition is actually rather involved. At some distance from QCP, $\Sigma(k,\omega) \approx \Sigma(k)$. However, the

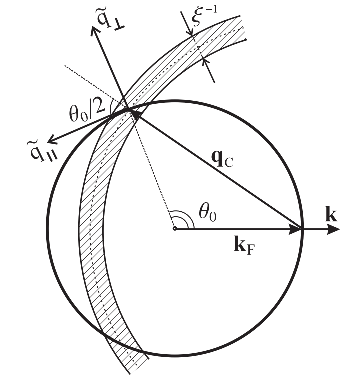

FIG. 1: The solid circular line represents the Fermi surface. The ring of radius q_c and width ξ^{-1} represents the effective interaction (2) via a soft bosonic mode. A fermion with the momentum $\boldsymbol{k}$ close to $\boldsymbol{k}_F$ strongly interacts with the two "hot spots" obtained by intersection of the Fermi circle and the interaction ring. The vector components $\tilde{q}_\perp$ and $\tilde{q}_\parallel$ are perpendicular and parallel to the Fermi surface at the hot spots.

frequency dependence of V, generated by the Landau damping, makes $\Sigma(k,\omega)$ predominantly ω-dependent in the immediate vicinity of QCP. We show that, when the fermion-boson interaction g is smaller than the Fermi energy $E_F \sim v_F k_F$, the crossover from $\Sigma(k,\omega) \approx \Sigma(k)$ to $\Sigma(k,\omega) \approx \Sigma(\omega)$ is separated from the weak-to-strong coupling crossover near QCP. The latter occurs when the dimensionless coupling $\lambda \sim (g/E_F)(\xi k_F) \propto \xi$ becomes of the order of 1. On the other hand, the crossover from $\Sigma(k)$ to $\Sigma(\omega)$ occurs at a small $\lambda \sim (g/E_F)^{1/2} \ll 1$, where Σ is still small, and $\partial_k \Sigma$ does not reach $-v_F$. Once $\Sigma(k,\omega)$ becomes $\Sigma(\omega)$, only $\partial_\omega \Sigma$ matters, i.e., $m^* = (Z v_F)^{-1}$ diverges with Z^{-1} at QCP, but not earlier. We present calculations in the 2D case, but the results are qualitatively valid also in the 3D case.

Momentum-dependent self-energy $\Sigma(k)$ away from QCP. In the Hartree-Fock approximation, the exchange diagram with the effective interaction (2) gives

$$\Sigma(\boldsymbol{k},\omega) = \int \frac{d\Omega\, d^2q}{(2\pi)^3}\, G(\boldsymbol{k}+\boldsymbol{q},\Omega+\omega)\, V(\boldsymbol{q}) = \int \frac{d^2q}{(2\pi)^2}\, n_F(\varepsilon_{\boldsymbol{k}+\boldsymbol{q}})\, V(\boldsymbol{q}), \tag{3}$$

where $n_F(\varepsilon)$ is the Fermi distribution function. The integration over $\boldsymbol{q}$ in Eq. (3) is restricted by the conditions that the vector $\boldsymbol{k}+\boldsymbol{q}$ lies inside the Fermi circle and the vector $\boldsymbol{q}$ belongs to the ring of radius q_c and width ξ^{-1} centered at the vector $\boldsymbol{k}$, as shown in Fig. 1. Clearly, Σ in Eq. (3) does not depend on ω, but it does depend of k, because the area of the ring inside the Fermi circle changes with k.

The derivative of Eq. (3) with respect to $\boldsymbol{k}$, taken at $\boldsymbol{k} = \boldsymbol{k}_F$, is given by the integral along the Fermi circle

$$\frac{\partial \Sigma}{\partial \boldsymbol{k}} = -\int \frac{d^2q}{(2\pi)^2}\, \delta(\varepsilon_{\boldsymbol{k}+\boldsymbol{q}})\, \boldsymbol{v}_{\boldsymbol{k}+\boldsymbol{q}}\, V(\boldsymbol{q}). \tag{4}$$

where $\delta(\varepsilon)$ is the Dirac delta-function, and $\boldsymbol{v}_{\boldsymbol{k}+\boldsymbol{q}}$ is the Fermi velocity at $\boldsymbol{k}+\boldsymbol{q}$. For large ξ, the integral (4) comes from the vicinity of the two "hot spots" $\boldsymbol{q}_c$ obtained by intersection of the Fermi circle and the circle of radius q_c centered at the point $\boldsymbol{k}_F$ on the Fermi circle (see Fig. 1). Decomposing the deviation from the hot spot $\tilde{\boldsymbol{q}} = \boldsymbol{q} - \boldsymbol{q}_c$ into $(\tilde{q}_\perp, \tilde{q}_\parallel)$, as shown in Fig. 1, and integrating over $\tilde{q}_\parallel$ first, we obtain $\partial_{\boldsymbol{k}} \Sigma = \hat{\boldsymbol{k}}\, \partial_k \Sigma$, where

$$\frac{\partial \Sigma}{\partial k} = -\lambda\, v_F \cos\theta_0, \qquad \frac{m^*}{m} = \frac{v_F}{v_F^*} = \frac{1}{1 - \lambda \cos\theta_0}. \tag{5}$$

Here θ_0 is the angle between $\boldsymbol{k}_F$ and $\boldsymbol{k}_F + \boldsymbol{q}_c$ in Fig. 1, such that $\sin(\theta_0/2) = q_c/(2k_F)$, and

$$\frac{\lambda}{2} = \frac{1}{v_F} \int \frac{d\tilde{q}_\parallel}{(2\pi)^2}\, V(\boldsymbol{q}) = \frac{g\,\xi}{4\pi v_F \cos(\theta_0/2)}. \tag{6}$$

We see that, if $\lambda \cos\theta_0 > 0$ (which implies $q_c < \sqrt{2} p_F$ for $\lambda > 0$), the effective mass increases with λ and nominally diverges at $\lambda \cos\theta_0 = 1$, while ξ is still finite, and QCP is not reached yet.

Crossover to the frequency-dependent self-energy $\Sigma(\omega)$. To verify whether Eq. (5) holds up to $\lambda \sim 1$, where m^* diverges, we need to go beyond the Hartree-Fock approximation and include the full fermionic and bosonic propagators and vertex corrections into the self-energy diagram. We assume and then verify that, for $g/E_F \ll 1$, the higher-order corrections predominantly renormalize V in Eq. (3), while vertex corrections and the renormalization of G can be neglected for arbitrary λ. The renormalization of $V(q)$ originates from the electron polarizability

$$\Pi(q,\Omega) = \int \frac{d^2k\, d\Omega}{(2\pi)^3}\, \frac{1}{i(\omega+\Omega) - \varepsilon_{\boldsymbol{k}+\boldsymbol{q}}}\, \frac{1}{i\omega - \varepsilon_{\boldsymbol{k}}} \tag{7}$$

via the relation $V^{-1}(q,\Omega) = V^{-1}(q) + \Pi(q,\Omega)$. For q near q_c, the static part of $\Pi(q,\Omega)$ comes from the fermions with high energies and we assume that it is already included into Eq. (2), which implies that ξ is the exact (renormalized) correlation length. The dynamical part of $\Pi(q_c,\Omega)$ comes from low energies and describes the Landau damping of the bosonic mode due to its decay into particle-hole pairs. For $\Omega \ll v_F q_c$, $\mathrm{Im}\Pi(q_c,\Omega) \propto |\Omega|/v_F^2$. Inserting the Landau damping term into Eq. (2), we find

$$V(q,\Omega) \approx \frac{g}{\xi^{-2} + (q-q_c)^2 + \gamma|\Omega|}, \qquad \gamma \sim \frac{g}{v_F^2}. \tag{8}$$

Re-evaluating $\Sigma(k,\omega)$ in Eq. (3) for the full $V(q,\Omega)$, we find that it now depends on both k and ω. Notice that causal analytical properties require that the interaction constant g in Eq. (8) must be positive: $g > 0$ [13].

We present the results for $\Sigma(k,\omega)$ first and discuss the details of calculations later. For small $\epsilon_k = v_F(k-k_F)$ and ω, we then obtain

$$\begin{aligned}\Sigma(k,0) &= -\lambda\cos\theta_0\,\varepsilon_k\,h_k(\eta), &(9)\\ \Sigma(k_F,\omega) &= -\lambda\,i\omega\,h_\omega(\eta), &(10)\\ \eta &= \gamma E_F\xi^2 \sim \lambda^2\,(E_F/g). &(11)\end{aligned}$$

Here and below we subtracted the renormalization of the chemical potential from $\Sigma(k,\omega)$, i.e., redefined $\Sigma(k,\omega)\equiv\Sigma(k,\omega)-\Sigma(k_F,0)$. The functions $h_k(\eta)$ and $h_\omega(\eta)$ have the following asymptotic behavior. For $\eta\ll 1$, $h_k(\eta)=1+O(\eta)$ and $h_\omega(\eta)\sim\eta\ln(1/\eta)$, i.e., the momentum-dependent piece in Σ almost coincides with Eq. (5), while the frequency dependence of Σ is weak. This is natural, because small η corresponds to small bosonic damping γ. However, for $\eta\gg 1$, we find the opposite behavior: $h_k(\eta)\propto\eta^{-1/2}\ll 1$ and $h_\omega(\eta)=1+O(\eta^{-1/2})$. In this case, the momentum dependence of Σ is weak compared with the Hartree-Fock approximation, while its frequency dependence is strong. Moreover, Σ does *not* depend on γ explicitly. Thus, the limiting forms of $\Sigma(k,\omega)$ are

$$\Sigma(k,\omega)\;\approx\;\begin{cases}-\lambda\cos\theta_0\,\varepsilon_k, & \eta\ll 1,\\ -\lambda\,i\omega, & \eta\gg 1.\end{cases} \tag{12}$$

When the system approaches QCP, and ξ increases, the parameter $\eta\propto\xi^2$ changes from $\eta\ll 1$ to $\eta\gg 1$. The crossover between the two asymptotic limits in Eq. (12) takes place at $\eta\sim 1$, which corresponds to

$$\lambda\sim\lambda_{\rm cr}=\sqrt{g/E_F}\ll 1. \tag{13}$$

Thus, the upper line in Eq. (12) stops being applicable already at $\lambda\ll 1$, before λ can generate a divergence in Eq. (5). In the vicinity of QCP, the lower line in Eq. (12) applies, and, instead of Eq. (5), we find

$$\frac{m^*}{m}\approx\frac{1}{Z}\approx 1+\lambda. \tag{14}$$

We see therefore that the effective mass in Eq. (14) diverges only at QCP, where $\xi\to\infty$, but not before, contrary to the conclusion one could draw from the Hartree-Fock approximation. This is the central result of the paper. Notice that the requirement $g>0$, mentioned after Eq. (8), guarantees that $Z\le 1$, because $\mathrm{sgn}(\lambda)=\mathrm{sgn}(g)$.

Further, since $\lambda_{\rm cr}\ll 1$, vertex corrections and renormalization of the fermionic G in Eq. (3) are small at $\lambda\sim\lambda_{\rm cr}$ and can be safely neglected. This justifies our approximation of including only the renormalization of the bosonic propagator. Moreover this approximation actually remains valid even at larger $\lambda\gtrsim 1$. Indeed, the modifications to Eq. (3) due to vertex corrections and residual momentum dependence of Σ are small in the parameter $\sqrt{g/E_F}\ll 1$ and can be safely neglected even when $\lambda=O(1)$. Although the fermionic $\Sigma(\omega)$ is not small at $\lambda=O(1)$, using the renormalized Green's function $G^{-1}(k,\omega)=i\omega(1+\lambda)-v_F(k-k_F)=Z[i\omega-v_F^*(k-k_F)]$ in Eq. (3) does not modify the lower line in Eq. (12), because the extra factor Z and the renormalization of $v_F^*=k_F/m^*$ compensate each other [14]. Similarly, the coefficient γ in Eq. (8) does not change, because the factor Z^2 coming from the two Green's function in the polarization bubble (7) compensates the renormalization of the factor $1/v_F^2\to 1/(v_F^*)^2$ in the expression (8) for γ. This behavior is typical for the Migdal-Eliashberg-type theories [15, 16].

Anomaly in the calculation of self-energy. Now we present details of the self-energy calculation and also explain why $\lambda_{\rm cr}$ vanishes if the fermionic bandwidth ($\sim E_F$) is set to infinity. Linearizing the fermionic dispersion near the two hot spots, we introduce $\zeta=v_F\tilde q_\perp$ and $\tilde\epsilon_k=(\boldsymbol{k}-\boldsymbol{k}_F)\cdot\boldsymbol{v}_{\boldsymbol{k}_F+\boldsymbol{q}_c}=\varepsilon_k\cos\theta_0$, where the vector $\boldsymbol{k}_F$ is selected parallel to $\boldsymbol{k}$ (see Fig. 1). Then $\Sigma(k,\omega)\equiv\Sigma(k,\omega)-\Sigma(k_F,0)$ is

$$\begin{aligned}\Sigma(k,\omega) &= -(i\omega-\tilde\epsilon_k)\,I(k,\omega), &(15)\\ I(k,\omega) &= \int\frac{d\Omega\,d\zeta\;\;\tilde V(\Omega,\zeta)}{2\pi\,[i(\omega+\Omega)-\tilde\epsilon_k-\zeta]\,(i\Omega-\zeta)}, &(16)\end{aligned}$$

where we introduced $\tilde V(\Omega,\zeta)$ similarly to Eq. (6)

$$\tilde V(\Omega,\zeta)=\frac{2}{v_F}\int\frac{d\tilde q_\parallel}{(2\pi)^2}\,V(\tilde q_\perp,\tilde q_\parallel,\Omega)=\frac{\lambda}{\sqrt{1+\gamma|\Omega|\xi^2}}. \tag{17}$$

Notice that, to this accuracy, $\tilde V(\Omega,\zeta)$ does not depend on ζ, i.e., $\tilde V(\Omega,\zeta)=\tilde V(\Omega)$.

The evaluation of $I(k,\omega)$ in the limit $k\to k_F$ and $\omega\to 0$ requires care, because the integrand in Eq. (16) contains two closely located poles separated by ω and $\tilde\epsilon_k$. If we approximate $\tilde V(\Omega)$ by a constant $\tilde V(0)=\lambda$, then, nominally, the integral (16) is ultraviolet-divergent and depends on the order of integration over Ω and ζ.

To evaluate the integral correctly, one must keep in mind that Eq. (16) is approximate, and higher-order terms in $(\boldsymbol{q}-\boldsymbol{q}_c)$ in G and V always make the integral over $\boldsymbol{q}$ convergent at $q-q_c\sim k_F$. If $\gamma=0$ in Eq. (17), then the integral over Ω must be taken first, because its convergence is provided only by the fermion Green's functions in Eq. (16). In this case, we obtain $I(k,\omega)=\lambda\tilde\epsilon_k/(i\omega-\tilde\epsilon_k)$ and $\Sigma(k,\omega)=\Sigma_k=-\lambda\tilde\epsilon_k$, reproducing the top line of Eq. (12).

On other hand, if γ is large in Eq. (17), then $\tilde V(\Omega)$ strongly depends on Ω and provides convergence of the integral over Ω. In this case, it is appropriate to integrate over ζ first, over the region where the linearized expression (16) is valid. Taking the integral over ζ first, we obtain $I(k,i\omega)=\lambda i\omega/(i\omega-\tilde\epsilon_k)$ and $\Sigma(k,\omega)=-\lambda i\omega$, reproducing the bottom line of Eq. (12). Notice that, although the frequency dependence of $\tilde V(\Omega)$ is essential to determine the correct order of integrations, the strength γ of this dependence drops out from the final answer. This situation bears mathematical similarity to the chiral anomaly in quantum field theory [17, 18].

The crossover between these two cases takes place when the characteristic Ω in Eq. (17) becomes of the order of $\zeta \sim E_F$. Using the definition (11), we find that the cases of weak and strong frequency dependence correspond to $\eta \lesssim 1$ and $\eta \gtrsim 1$, as in Eq. (12).

The fact that the crossover occurs at $\eta \sim 1$, i.e., at $\lambda_{\rm cr} \ll 1$, is a consequence of $V(\boldsymbol{q},\Omega)$ being peaked on a *circle* $|\boldsymbol{q}| = q_c$. If $V(\boldsymbol{q},\Omega)$ were peaked at a given *vector* $\boldsymbol{q}_c$ in a crystal, then $\tilde{V}$ would have the conventional, Ornstein-Zernike form $\tilde{V}(\Omega,\tilde{q}_\perp) = \int d\tilde{q}_\parallel/(\xi^{-2} + \tilde{q}_\perp^2 + \tilde{q}_\parallel^2 + \gamma|\Omega|) \propto (\xi^{-2} + \tilde{q}_\perp^2 + \gamma|\Omega|)^{-1/2}$. In this case, the crossover takes place when all three terms become comparable: $\xi^{-2} \sim \tilde{q}_\perp^2 \sim \gamma|\Omega|$. Since typical $\Omega \sim v_F\tilde{q}_\perp$, the crossover in the vector case occurs at $\gamma v_F \xi \sim 1$, i.e., $\lambda_{\rm cr} \sim 1$ [15]. However, m^* does not diverge even when Σ remains $\Sigma(k)$ up to $\lambda \sim 1$, because the correction to velocity $\partial_{\boldsymbol{k}}\Sigma = -\lambda \boldsymbol{v}_{\boldsymbol{k}+\boldsymbol{q}_c}$ is not antiparallel to $\boldsymbol{v}_{\boldsymbol{k}}$ in the absence of nesting, so the magnitude of the Fermi velocity does not vanish for any finite λ [15].

For completeness, it is instructive to see how the crossover from the top to the bottom line in Eq. (12) happens if we always integrate over Ω first in Eq. (16). Let us deform the contour of integration over Ω to either upper or lower complex half-plane. For $-\tilde{\epsilon}_k < \zeta < 0$, each half-plane contains just one pole. Wrapping the contour around the pole and integrating over ζ within the specified limits, we obtain the Σ_k contribution to Σ. If $\tilde{V}$ does not depend on Ω, the calculation stops here. However, when $\tilde{V}(\Omega)$ depends on Ω and is given by Eq. (17), we also need to consider a contribution from the branch cut in $\tilde{V}(\Omega)$ along the imaginary axis of Ω where $\Omega = i\nu + \delta$ and $|\Omega| = i\nu\,\mathrm{sgn}\,\delta$. Evaluating the contribution from the branch cut and combining it with the contribution from the pole, we find $\Sigma = -\lambda\tilde{\epsilon}_k + (i\omega - \tilde{\epsilon}_k)I_{\rm bc}$, where

$$\begin{aligned} I_{\rm bc} &= (2/\pi)\int_0^{E_F} d\zeta \int_0^\infty d\nu\, \mathrm{Im}\tilde{V}(i\nu)/(\nu+\zeta)^2 \\ &= \frac{2}{\pi}\int_0^\eta dz \int_0^\infty \frac{d\varpi}{(\varpi+z)^2}\,\mathrm{Im}\left(\frac{\lambda}{\sqrt{1-i\varpi}}\right). \end{aligned} \quad (18)$$

Here we introduced dimensionless variables $z = \zeta\gamma\xi^2$ and $\varpi = \nu\gamma\xi^2$. For $\eta \ll 1$, Eq. (18) gives a small $I_{\rm bc} \sim \lambda\eta\ln(1/\eta)$. In the opposite limit $\eta \gg 1$, the integral over z can be extended to infinity, and the integral over ϖ yields $I_{\rm bc} = \tilde{V}(0) = \lambda$ via the Kramers-Kronig relation. Notice that this result does not depend explicitly on the detailed form of the frequency dependence in $\tilde{V}(\Omega)$ as long as the integral (18) quickly converges and can be extended to infinity. Substituting $I_{\rm bc}$ into Σ, we reproduce both lines in Eq. (12) for $\eta \ll 1$ and $\eta \gg 1$.

Conclusions. In this paper, we studied the quantum critical behavior of an isotropic system of fermions near a $T = 0$ transition into a density-wave state with a finite momentum q_c. We demonstrated that, upon approaching QCP, the fermionic self-energy crosses over from $\Sigma(k,\omega) \approx \Sigma(k)$ to $\Sigma(k,\omega) \approx \Sigma(\omega)$. We showed that the crossover occurs while the dimensionless coupling λ (which diverges at QCP) is still small. We found that the effective mass remains finite and positive away from QCP, and diverges only at QCP as $m^* \propto 1/Z \propto 1+\lambda$.

Our results apply to both charge- and spin-density-wave instabilities. In the latter case, spin-orbital interaction generally induces anisotropy in the spin space, e.g., for easy axis, the interaction in Eq. (1) is mediated by z component of spins. This only affects the numerical coefficient in Σ, proportional to the number of fluctuating spin components, but does not change the conclusions.

We thank D. Maslov for useful discussions. The work was supported by the NSF Grant DMR-0240238 (AVC); by US-ONR, LPS, and DARPA (VMG); and by the NSF Grant DMR-0137726 (VMY).

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